Graph-based Methods for Connectivity Analysis

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Brain Connectivity

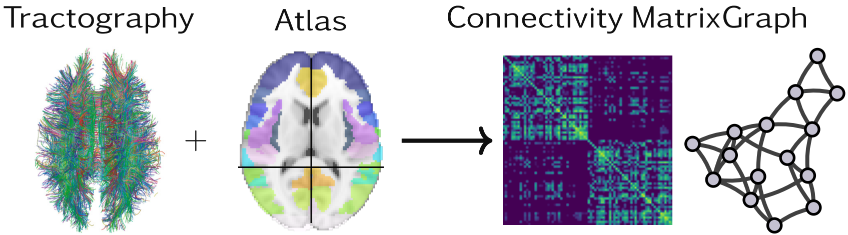
Graph Signal Processing

Graph Curvature

Non Invasive Imaging

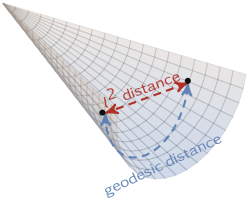
Figure: made by DinosoftLabs

Connectivity Analysis

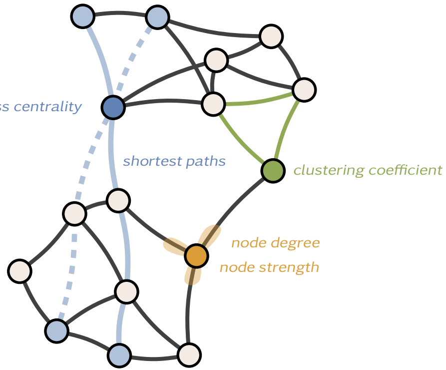


- Establish a map of Human Cognition
- Perform diagnosis or prevention based on imaging
- Follow pathologies or treatments

Connectome Comparison



Matrix Distance

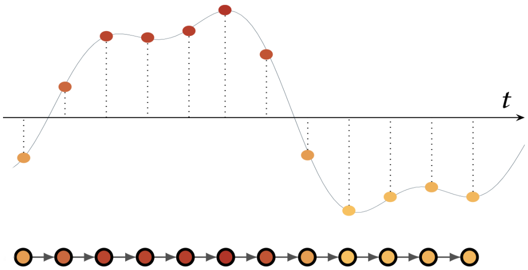


Graph Markers

Issues, and our contribution

- Local *vs* global
- Combinatorial *vs* connectivity information

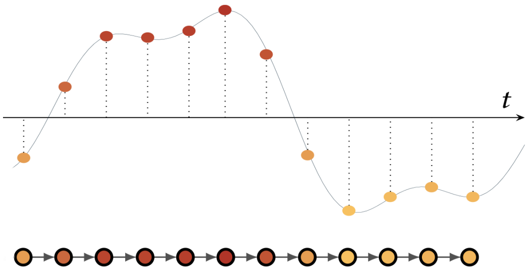
Graph Signal Processing

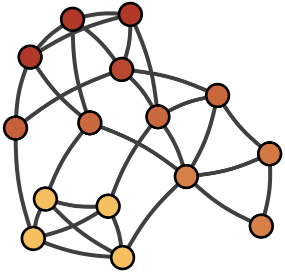


gradient ∇ Laplacian L

$$s^\top L s = \sum_{i,j} w_{i,j} \left(s_i - s_j\right)^2$$

Graph Signal Processing

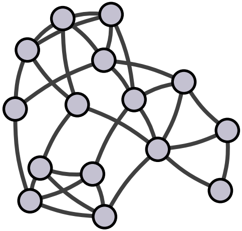


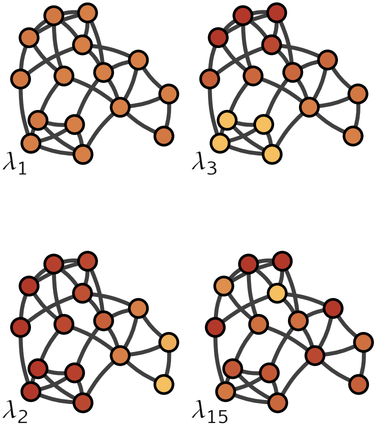


gradient ∇ Laplacian L

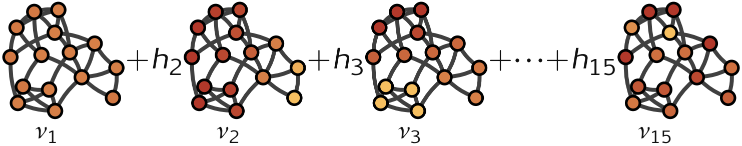
$$s^{\mathsf{T}} L s = \sum_{i,j} w_{i,j} \left(s_i - s_j \right)^2$$

Fourier Transform





$$s = \chi h = h_1 \quad + h_2 \quad + h_3 \quad + \cdots + h_{15}$$



A Wasserstein Distance for Graphs

Smooth signals $\mathcal{G} \mapsto \mathcal{N}\left(0, L^{-1}\right)$ Maretic et al. (2019)

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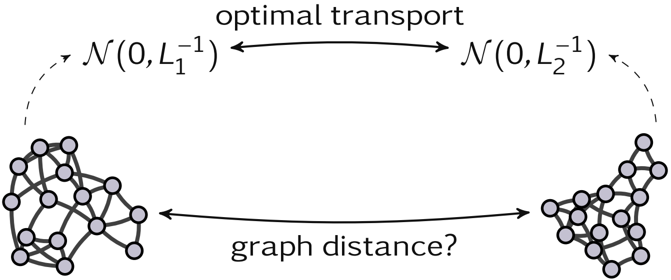
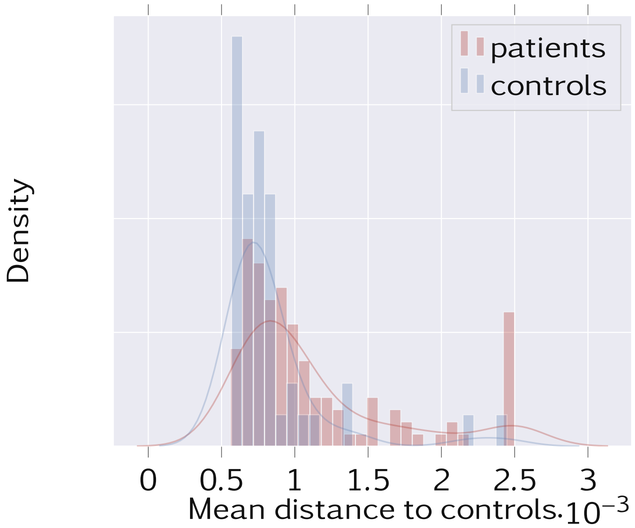
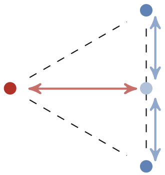


Comparing Two Cohorts with this Distance



Barycenter

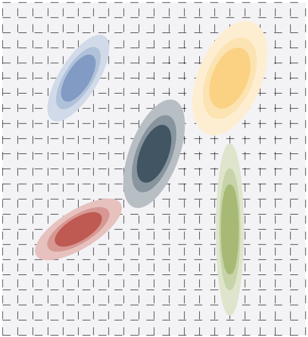


- Optimal transport theory

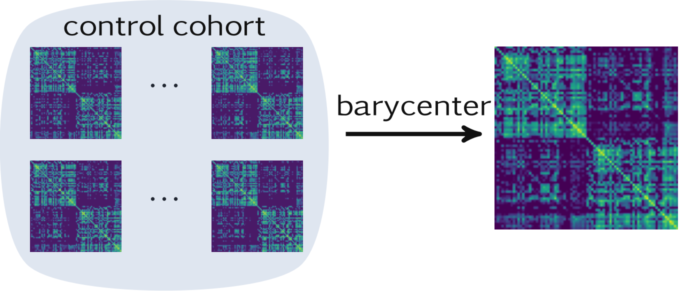
$$G(\mu) = \mu$$

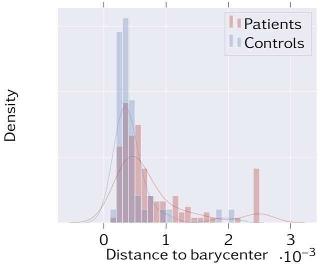
Álvarez-Esteban et al. (2015)

- Tractable iteration for Gaussians measures

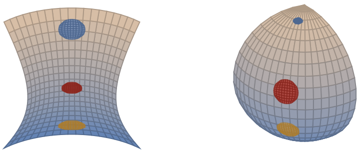


Barycenter of Connectivity Matrices



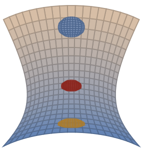


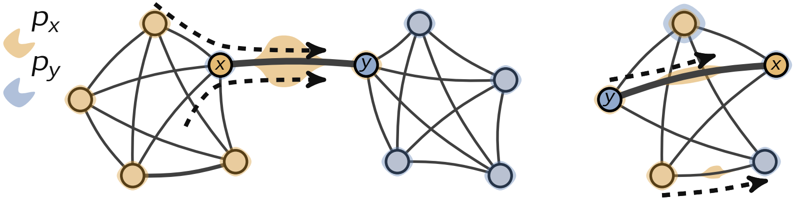
Graph Curvature



$$W_1(p_x, p_y) = (1 - \kappa(x,y))d(x,y)$$

Graph Curvature





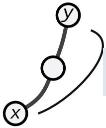
$$W_1(p_x, p_y) = (1 - \kappa(x,y))d(x,y)$$

Metric space for connectivity maps

$$\kappa(x,y) = 1 - \frac{W_1(p_x, p_y)}{d(x,y)}$$

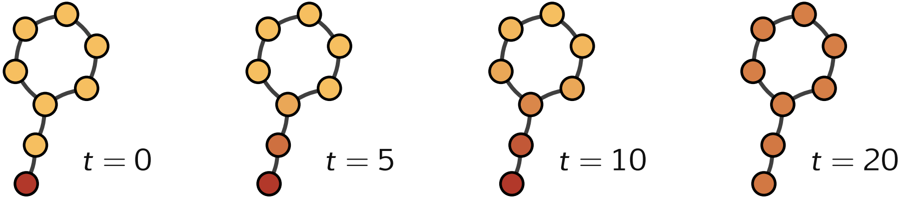
Network curvature as a hallmark of brain structural connectivity
Farooq et al. (2019)

$$d(x,y) = \texttt{shortest\_path}(x,y)$$



$$d(x,y) = 2$$

Heat Kernel Distance

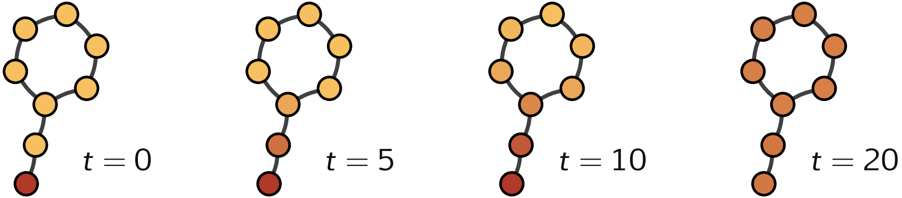


$$\left(\frac{\partial}{\partial t} + L\right) f(t) = 0 \qquad f(t) = \underbrace{e^{-tL}}_{k_t} f_0$$

$k_t(x,y)$ amount of heat transfered from x to y in time t

$$d_t(x,y) = k_t(x,x) + k_t(y,y) - 2k_t(x,y)$$

Heat Kernel Distance



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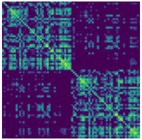
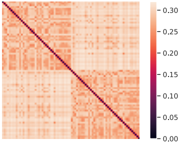
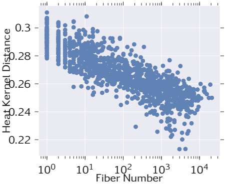
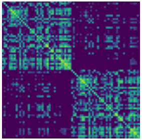
Heat Kernel Distance and Curvature

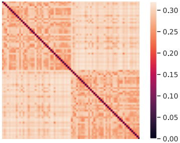


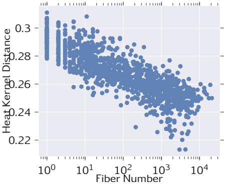


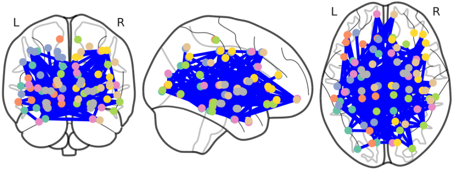


Heat Kernel Distance and Curvature

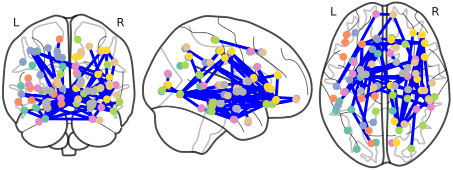








Curvature Difference



Strength Difference

Conclusion

- Applied a new tool based on GSP and Optimal Transport
- Developped a notion of barycenter of connectomes
- Investigated a new formulation of connectome curvature